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Pion production spectrum at the atmosphere in the energy range 3–100 GeV

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The pion production spectrum in the atmosphere is derived by using the recent parameters on the high energy inelastic interactions (pp) from the machine experiments and by using CE model from the satellite data on the primary spectrum of Grigorov *et al* (1971) which is in accord with that of Allen & Apostolakis (1961), Coates & Nash (1962), Judge & Nash (1965). Pion production spectrum assumes the form $P(E_\pi)dE_\pi = 0.34E_\pi^{-2.8}dE_\pi$ in the pion energy range 3-100 GeV.

1. Introduction

Recently Hook & Turver (1974) have reviewed the spectrum of primary cosmic radiation. The only satellite data among them were found from the measurement of Grigorov *et al* (1971). The satellite data are more reliable, since these data are almost free from the loss of intensity due to nucleon-air-nucleus collisions and the geomagnetic effects of protons at high latitudes are almost negligible for protons in the energy above 10 GeV. For primary proton of energy E_p(GeV) the energy spectrum determined by Grigorov *et al* (1971) can be represented by the relation

$$N(E_p)dE_p = BE_p^{-\gamma}dE_p \text{ cm}^{-2}.\text{sec}^{-1}\ Sr^{-1}\,\text{GeV}^{-1}. \quad \ldots (1)$$

where $B = 2.54$ and $\gamma = 2.6$.

The cosmic pion production spectrum has been derived from the experimental results of primary spectrum and by the use of CE model of nucleon air-nucleus collisions. The collision parameters governing mainly the calculation of the secondary spectrum have been given by : K_T (nucleon inelasticity) $= 0.422$, K_π (pion inelasticity) $= 0.292$ and charged pion multiplicity due to pp collisions $\langle n_s \rangle = 1.80E_p^{1/4}$.

2. Theoretical Aspects

The number of charged pions (one sign) produced in the forward direction in the C system, $N(E_\pi)$ is given by Cocconi *et al* (1961)

$$N(E_\pi)dE_\pi = \frac{A}{T_p} \exp \left(-\frac{E_\pi}{T_p}\right) dE_\pi, \quad \ldots (2)$$

where E_π is the pion energy in L system, A is the mean pion multiplicity of one sign emitted in the forward direction on the C system and T_p is the mean pion energy. The quantities A, T_p and K_π are related by the equation

$$AT_p = \frac{1}{3} K_\pi E_p, \qquad \ldots \quad (3)$$

where E_p is the primary energy. The accelerator data on the average charged pion multiplicity $<n_s>$ for p-low Z nucleus inelastic interactions in general follow the Fermi distribution, viz.,

$$<n_s> = 1.80 E_p^{1/4},$$

which shows that $A = 0.45 E_p^{1/4}$, where E_p is expressed in GeV units.

Brooke *et al* (1964) have assumed in the Constant Energy (to be referred to as *CE*) model that pions are emitted with equal energy in C system, half being in the forward direction and half in the backward direction and the energy of the fast pions in the L system is taken as $K_\pi E_p/3A$. If one assumes the nucleon inelasticity K_T and pion inelasticity K_π as constants for each interaction, the production spectrum of charged pions can be found from *CE* model by using the relation

$$P(E_\pi)dE_\pi = \frac{2}{[1-(1-K_T)^{\gamma-1}]} \frac{B}{(1-\alpha)} a^u \left(\frac{K_\pi}{3}\right)^v E_\pi^{w} dE_\pi,$$

where $u = (2-\gamma)/(1-\alpha)$, $v = (\gamma-\alpha-1)/(1-\alpha)$, $w = (2\alpha-\gamma)/(1-\alpha)$ and $\alpha = 1/4$.

Hagedorn & Ranft (1958) have calculated the values of partial inelasticities K_q using statistical models for pp collisions at momenta 12.5, 18.8, 30 and 300 GeV/c. O'Brien (1969) interpolated the values of K_q and obtained the values of partial inelasticities presented in table 1.

Table 1

q	K_q
n	0.211
n	0.211
π^+	0.180
π^-	0.112
π^0	0.180
K^+	0.034
K^-	0.022
K^0	0.034

3. Results

By taking $K_T = 0.422$ for $q = p, n$, the pion production spectrum can be expressed by the relation

$$P(E_\pi)dE_\pi = 3.11 K_\pi^{1.8} E_\pi^{-2.8} dE_\pi$$

For $q = \pi\pm$, $K_q = 0.292$ we obtain the pion production spectrum in the atmosphere which is of the form

$$P(E_\pi)dE_\pi = 0.34 E_\pi^{-2.80} dE_\pi.$$

Table 2 shows the production spectra of charged pions derived by different authors from the sea level muon spectra (experimental) by using the conventional one dimensional pion atmospheric diffusion equation. The production spectrum of charged pions found by different authors can be expressed as

$$P(E_\pi)dE_\pi = A\ E_\pi^{-\gamma_\pi} dE_\pi,$$

where E_π is expressed in GeV and intensities in units of cm^{-2} sec^{-1} sr^{-1} GeV^{-1}.

Table 2. The values of A_π and γ_π found by different authors

Authors		γ_π
Allen & Apostolakis (1961)	0.299	2.84
Coates & Nash (1962)	0.325	2.88
Judge & Nash (1965)	0.373 ± 0.04	2.97 ± 0.14
Baber *et al* (1967)	0.24	2.65
Crookes & Rastin (1973)	0.26	2.73
Bhattacharyya (1974)	0.22 ± 0.04	2.65 ± 0.11
Bhattacharyya *et al* (1974)	0.23	2.68
Present work	0.34	2.80

4. Discussions

Thus using the recent data on the primary spectrum along with the machine data on the p-low Z inelastic interactions when fitted to the CE model, one obtains the pion production spectrum which is in accordance with that of Allen & Apostolakis (1961), Coates & Nash (1962) and Judge & Nash (1965). The values of K_T and K_π were 0.422 and 0.292, respectively, and these were much lower than those of Brooke *et al* (1964) which were 0.47 and 0.35, respectively. The production spectrum of pions derived in the present work agrees well with the sea level experimental muon spectrum in the pion energy range 3-100 GeV. The difference between K_T and K_π is 0.13 which agrees with the best estimated value 0.12±0.04 of Brooke *et al* (1964).

5. Conclusion

The *CE* model has been used to derive the energy spectrum of cosmic pions in the atmosphere in the energy range 3-100 GeV which is in agreement with the derived results of Allen & Apostolakis (1961), Coates & Nash (1962) and Judge & Nash (1965).

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